

# Born-Infeld Thin-shell Wormholes Supported by Generalized Cosmic Chaplygin Gas

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## Abstract

This paper investigates thin-shell wormholes in Born-Infeld theory supported by generalized Cosmic Chaplygin gas (GCCG). We study their stability via radial perturbations for distinct values of charge and Born-Infeld parameter. The comparison of wormhole solutions corresponding to generalized Chaplygin gas, modified Chaplygin gas with GCCG equation of state is established. It is found that similar type of wormhole solutions exists for small value of charge and Born-Infeld parameter for all type of equation of state, while some extra stable as well as unstable solution are found corresponding to large value of charge and Born-Infeld parameter. Thus, it is concluded that GCCG and large value of charge may be responsible for such extra solutions.

**Keywords:** Thin-shell wormholes; Born-Infeld electrodynamics; Stability.

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## 1 Introduction

A wormhole (WH) is a hypothetical object in spacetime which behaves like a smooth bridge between two different universes or smooth shortcut between

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remote parts of a single universe (Visser, 1989). Recently, WH physics has been taken as hot cake due to its interesting features. The first traversable WH was obtained by Morris and Thorn (1988) with two asymptotically flat regions which are joined by a minimal surface area known as WH throat. For WH to be traversable, it is necessary that the WH throat must satisfies the flare out condition (Lobo, 2008), i.e., matter threading the WH throat should violate the null energy condition. Such matter is known as exotic matter. However, physical viability of such WHs was a big issue. In this scenario, Visser (2003) showed that the amount of exotic matter located around the throat can be reduced by taking an appropriate choice of WH geometry.

It is well known that the existence of exotic matter (violation of null energy condition) is always accompanied by WH solutions. In this context, one can construct thin-shell WH through cut and paste technique to confine the exotic matter at WH throat (Lobo, 2008). This is an elegant and efficient procedure used to minimize the violation of null energy condition through thin-shell formalism. Poisson and Visser (1995) were the pioneer who constructed Schwarzschild thin-shell WH and investigated its stability through radial perturbations. After that, many authors (Lobo, 2005; Lemos and Lobo, 2008; Rahaman et al. 2009; Goncalo, 2010; Rahaman et al. 2012) have constructed thin-shell WHs with this procedure and used Darmois-Israel junction conditions (Darmois, 1927; Israel, 1966, *ibid.* 1967) to explore WH dynamics. In recent years, some authors (Eiroa and Simeone, 2005; Eiroa, 2008; Rahaman et al. 2010; Bejarano and Eiroa, 2011) constructed spherical thin-shell WHs in this theory and investigated their linearized stability under radial perturbations.

The selection of equation of state (EoS) for dynamical analysis of matter present at the shell has an important role in the existence and stability of WH solutions. Thus, many authors have taken account different EoS in search of viable thin-shell WHs. In this context, family of Chaplygin gas (Chaplygin (1939); Von Karman (1941)) has been used successfully in describing various astronomical phenomenon (wormholes, cosmological evolution of the early and present Universe). Eiroa and Simeone (2007) found stable static WH solutions with Chaplygin gas corresponding to fixed values of parameters. Later on, Bandyopadhyay et al. (2009) carried out this work with simple modified Chaplygin gas (MCG) EoS and found some more stable WH solutions. Eiroa (2009) have constructed thin-shell WHs numerically supported with generalize Chaplygin gas (GCG) and shows that some extra solution can exists. This indicates that choice of EoS may play a significant role

in the existence of WH solutions. Also, Gorini et al. (2008, 2009) found WH like solutions by using Chaplygin gas and GCG. We have also studied thin-shell WHs with family of Chaplygin gas and found distinct solutions corresponding to distinct EoS (Sharif and Azam, 2013a, 2013b, 2013c).

Since GCCG is less constrained as compared to MCG and GCG and is capable of adapting itself to any domain of cosmology, depending upon the choice of parameters. Thus it has a more universal character and the big-rip singularity can easily be avoided in this model. For instance, many authors have used GCCG to; discuss the evolution of the universe from dust era to  $\Lambda$ CDM (Chakraborty et al. 2007), studied the background dynamics of GCCG in brane world gravity (Rudra, 2012), presented a singularity free model for an expanding universe undergoing a late acceleration (Ratul et al. 2013), studied the role of GCCG in accelerating universe (Prabir, 2013), discussed FRW universe in loop Quantum gravity with GCCG as dark energy candidate (Ranjit and Debnath, 2014), studied GCCG inflationary universe model for a flat FRW geometry (Sharif and Rabia, 2014), found stable traversable WHs (Sharif and Jawad, 2014), found stable and unstable Schwarzschild de-sitter and anti-de-sitter thin-shell WHs (Sharif and Mumtaz, 2014).

The nonlinear electrodynamics (NED) theory which is considered as the most outstanding viable theory among all the NED theories introduced by Born-Infeld (BI) in 1934 (Born and Infeld, 1934). It has a therapeutic power for singularities appear in Maxwell theory. Hoffmann (1935) found spherically symmetric solution by coupling general relativity with BI electrodynamics theory which describes the gravitational field of a charged object. It was shown that Maxwell and BI theories possess the property of duality invariance like electric and magnetic fields (Gibbons and Rasheed, 1995). The use of BI action in the low energy string theory has been interesting to study such NED theories (Fradkin and Tseytlin, 1985; Bergshoeff et al. 1987; Metsaev et al. 1987; Tseytlin, 1997; Brecher et al. 1998). It was argued that trajectories of photons are not null geodesics of the background metric in curved spacetimes within BI electrodynamics (Plebanski, 1970) but rather follows null geodesics of a physical geometry influenced by the nonlinearities of electromagnetic field. Breton (2002) examined the geodesic structure of BI black holes.

The Born-Infeld action in four-dimension associated with Einstein gravity

is given by

$$\mathcal{S} = \int d^4x \sqrt{g} \left( \frac{R}{16\pi} + L_{BI} \right), \quad (1)$$

where  $g$ ,  $R$  and  $L$  correspond to the determinant of the metric tensor, Ricci scalar and non-linear Lagrangian coupled with electromagnetic field tensor defined as

$$L_{BI} = \frac{1}{4\pi b^2} \left( 1 - \sqrt{1 + \frac{1}{2} F_{\sigma\nu} F^{\sigma\nu} b^2 - \frac{1}{4} {}^*F_{\sigma\nu} F^{\sigma\nu} b^4} \right), \quad (2)$$

where  $F_{\sigma\nu} = \partial_\sigma A_\nu - \partial_\nu A_\sigma$  and  ${}^*F_{\sigma\nu} = \frac{1}{2} \sqrt{-g} \epsilon_{\gamma\delta\sigma\nu} F^{\gamma\delta}$  are electromagnetic field tensor and Hodge dual of  $F_{\sigma\nu}$ , respectively and  $\epsilon_{\gamma\delta\sigma\nu}$  is the Levi-Civita symbol. The value of BI parameter  $b$  will make a comparison between Born-Infeld and Maxwell electrodynamics, i.e., BI Lagrangian will approach to Maxwell Lagrangian in the limit  $b \rightarrow 0$ . The variation of action with respect to  $g_{\mu\nu}$  and  $A_\nu$  yields Einstein field equations whose solution corresponds to vacuum spherically symmetric solution (Gibbons and Rasheed, 1995; Breton, 2002) given by

$$ds^2 = -H(r)dt^2 + H^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (3)$$

with

$$H(r) = 1 - \frac{2M}{r} + \frac{2}{3b^2} \left\{ r^2 - \sqrt{r^4 + b^2 Q^2} + \frac{\sqrt{|bQ|^3}}{r} F \left[ \arccos \left( \frac{r^2 - |bQ|}{r^2 + |bQ|} \right), \frac{\sqrt{2}}{2} \right] \right\}, \quad (4)$$

where elliptic integral of the first kind  $F(\gamma, k)$  is defined by

$$F(\gamma, k) = \int_0^{\sin\gamma} [(1-y^2)(1-k^2y^2)]^{-\frac{1}{2}} dy = \int_0^{\sin\gamma} (1-k^2 \sin^2\phi)^{-\frac{1}{2}} d\phi,$$

and  $M$ ,  $Q$  represent mass and charge of the BI black hole. The horizons of (3) can be found numerically by setting  $H(r) = 0$ . A regular event horizon is obtained for a given value of  $b$  and small value of charge, i.e.,  $0 \leq \frac{|Q|}{M} \leq \omega_1$ , where  $\omega_1 = \left(\frac{9|b|}{M}\right)^{\frac{1}{3}} [F(\pi, \frac{\sqrt{2}}{2})]^{-\frac{2}{3}}$ . For  $\omega_1 < \frac{|Q|}{M} < \omega_2$ , there exist two regular horizons inner and outer similar to Reissner-Nordström geometry. However, for  $\frac{|Q|}{M} = \omega_2$  and  $\frac{|Q|}{M} > \omega_2$ , there exists one degenerate horizon and naked singularity respectively, where  $\omega_2$  can be obtained numerically through the

condition  $H(r) = 0 = H'(r)$  (see details (Eiroa and Aguirre, 2012)). It is easy to check that in the limit  $b \rightarrow 0$  (Reissner-Nordström case),  $\omega_1 = 0$  and  $\omega_2 = 1$ .

Recently, much interest in non-linear electrodynamics theories has been aroused in application to WH geometries and cosmological phenomena (Mazharimousavi and Halilsoy, 2015; Gullu et al. 2015; Jana and Kar, 2015; Gullu et al. 2015; Mazharimousavi et al. 2013). In this scenario, Baldovin et al. (2000) showed that a certain field configuration in Born-Infeld electromagnetism in flat spacetime can be interpreted as a WH. Arellano et al. (2009) studied some properties for the evolving WH solutions in non-linear electrodynamics. Richarte and Simeone (2009, 2010) studied the spherically symmetric thin-shell WHs in the scenario of Born-Infeld gravity and analyzed the mechanical stability of WH configurations. Rahaman et al. (2010) construct and discuss various aspects of thin-shell WHs from a regular charged black hole in the framework of non-linear electrodynamics. Eiroa and Simeone (2011) investigated the mechanical stability of thin-shells both in Einstein Maxwell and Born Infeld theory. Mazharimousavi et al. (2011) used the Hoffman-Born-Infeld Lagrangian to construct the black holes and viable thin-shell WHs. In particular, they investigate the stability of thin-shell WHs supported by normal matter. Halilsoy et al. (2014) constructed thin-shell WHs from the regular Hayward black hole with linear, logarithmic, Chaplygin etc., equation of states and found that Hayward parameter makes thin-shell WHs more stable.

In this work, we have construct BI thin-shell WHs and investigate their stability supported with GCCG. We have compared our results with recent work supported with GCG (Eiroa and Aguirre, 2012) and MCG (Sharif and Azam, 2014). The paper is planned as: Section **2** deals with the basic equations for the construction of spherical thin-shell WHs with GCCG. In section **3**, we present the general procedure to investigate stability of static WH solutions. In section **4**, we apply the general formalism developed in section **2** and **3** to construct and explore stability of BI thin-shell WHs. We summarize our results in the last section.

## 2 Thin-Shell Wormhole Construction: Fundamental Equations

In this section, we will work out basic equations for BI thin-shell WH with GCCG. We take two copies,  $\mathcal{S}^\pm = \{x^\mu = (t, r, \theta, \phi)/r \geq a\}$ , of the BI black hole (3) each with  $r \geq a$  in such a way that these geometries are prevented from horizons and singularities, where ‘ $a$ ’ is a constant. The joining of these geometries at the timelike hypersurfaces  $\Sigma = \Sigma^\pm = \{x^\mu/r - a = 0\}$ , to form a new manifold (geodesically complete)  $\mathcal{S}^\pm = \mathcal{S}^+ \cup \mathcal{S}^-$  representing a WH having two regions connected by a junction surface (throat of the WH).

For the dynamical analysis of this traversable WH, we shall adopt the Darmois-Israel formalism. This formalism is one of the basic formulation used to study the dynamics of the matter field located at the WH throat, providing a set of equations correspond to field equations. The synchronous timelike hypersurface which is throat of the WH (junction surface) is defined by the coordinates  $\varsigma^i = (\tau, \theta, \phi)$ . Thus the intrinsic metric to the hypersurface  $\Sigma$  can be written as

$$ds^2 = -d\tau^2 + a^2(\tau)(d\theta^2 + \sin^2\theta d\phi^2), \quad (5)$$

where  $\tau$  is the proper time. Following the Darmois-Israel formalism, the explicit expression for the extrinsic curvature (second fundamental forms) connected with two sides of the shell is defined as

$$K_{ij}^\pm = -n_\gamma^\pm \left( \frac{\partial^2 x_\pm^\gamma}{\partial \varsigma^i \partial \varsigma^j} + \Gamma_{\mu\nu}^\gamma \frac{\partial x_\pm^\mu}{\partial \varsigma^i} \frac{\partial x_\pm^\nu}{\partial \varsigma^j} \right), \quad (i, j = \tau, \theta, \phi), \quad (6)$$

where the superscripts  $\pm$  stands for exterior and interior geometry, respectively. The outwards unit 4-normals to  $\Sigma$  with  $n^\gamma n_\gamma = +1$ , are given by

$$n_\gamma^\pm = \pm \left| g^{\mu\nu} \frac{\partial f}{\partial x^\mu} \frac{\partial f}{\partial x^\nu} \right|^{-\frac{1}{2}} \frac{\partial f}{\partial x^\gamma} = \left( -\dot{a}, \frac{\sqrt{H(r) + \dot{a}^2}}{H(r)}, 0, 0 \right). \quad (7)$$

Using Eqs. (5) and (6), we obtain the non-vanishing components for geometry (3) as follows

$$K_{\tau\tau}^\pm = \mp \frac{H'(a) + 2\ddot{a}}{2\sqrt{H(a) + \dot{a}^2}}, \quad K_{\theta\theta}^\pm = K_{\phi\phi}^\pm = \pm \frac{1}{a} \sqrt{H(a) + \dot{a}^2}, \quad (8)$$

where prime and dot corresponds to  $\frac{d}{dr}$  and  $\frac{d}{d\tau}$ , respectively.

The surface stresses, i.e., surface energy density  $\sigma$  and surface pressures  $p = p_\theta = p_\phi$ , are determined by the surface stress-energy tensor  $S_{ij} = \text{diag}(\sigma, p_\theta, p_\phi)$  and Einstein equations or Lanczos equations on the shell are given by

$$S_{ij} = \frac{1}{8\pi} \{g_{ij}K - [K_{ij}]\}, \quad (9)$$

where

$$[K_{ij}] = K_{ij}^+ - K_{ij}^-, \quad K = \text{tr}[K_{ij}] = [K_i^i].$$

From Eqs. (8) and (9), the surface stresses of the shell turns out as

$$\sigma = -\frac{\sqrt{H(a) + \dot{a}^2}}{2\pi a}, \quad (10)$$

$$p = p_\theta = p_\phi = \frac{\sqrt{H(a) + \dot{a}^2}}{8\pi} \left[ \frac{2\ddot{a} + H'(a)}{H(a) + \dot{a}^2} + \frac{2}{a} \right]. \quad (11)$$

From the above equation, the negativity of surface energy density will insure the presence of exotic matter at the throat. To discuss the physical aspects of this exotic matter, we choose GCCG (González-Díaz, 2003) EoS because the free parameter in it can encompass different types of matter defined by

$$p = -\frac{1}{\sigma^\beta} [L + (\sigma^{1+\beta} - L)^{-\omega}], \quad (12)$$

where  $L = \frac{D}{1+\omega} - 1$ ,  $D \in (-\infty, \infty)$  and  $-D < \omega < 0$ . Here, we take  $D$  to be positive constant other than unity. Also, the above equation reduces to GCG in the limit  $\omega \rightarrow 0$ .

For the dynamical analysis of thin-shell WH, we develop a second order differential equation (equation of motion) by using Eqs.(10) and (11) in (12) given by

$$\begin{aligned} & \{ [2\ddot{a} + H'(a)] a^2 + [H(a) + \dot{a}^2] 2a \} [2a]^\beta - 2(4\pi a^2)^{1+\beta} [H(a) + \dot{a}^2]^{\frac{1-\beta}{2}} \\ & \times \left[ L + \left\{ (2\pi a)^{-(1+\beta)} (H(a) + \dot{a}^2)^{\frac{(1+\beta)}{2}} - L \right\}^{-\omega} \right] = 0. \end{aligned} \quad (13)$$

This equation provides full understanding of the thin-shell WH satisfied by the throat radius with GCCG in BI theory.

### 3 Stability Analysis of Static Solutions: A Standard Approach

In this section, we follow the standard approach to investigate the stability of static BI WH solutions with GCCG under radial perturbations (Bejarano and Eiroa, 2011). From Eqs.(10), (11) and (13), the static configuration of surface energy density, surface pressure and evolution equation of BI WH takes the form

$$\sigma_0 = -\frac{\sqrt{H(a_0)}}{2\pi a_0}, \quad p_0 = \frac{2H(a_0) + a_0 H'(a_0)}{8\pi a_0 \sqrt{H(a_0)}}, \quad (14)$$

$$\begin{aligned} & [a_0^2 H'(a_0) + 2a_0 H(a_0)] [2a_0]^\beta - 2(4\pi a_0^2)^{1+\beta} [H(a_0)]^{\frac{1-\beta}{2}} \\ & \times \left[ L + \left\{ (2\pi a_0)^{-(1+\beta)} (H(a_0))^{\frac{(1+\beta)}{2}} - L \right\}^{-\omega} \right] = 0. \end{aligned} \quad (15)$$

The law of conservation of energy on the WH throat can be defined with Eqs.(10) and (11) as

$$\frac{d}{d\tau}(\sigma\Omega) + p\frac{d\Omega}{d\tau} = 0, \quad (16)$$

where  $\Omega = 4\pi a^2$  is known as area of WH throat. This equation describes that the sum of rate of change of WH throat's internal energy and work done by the internal forces is equal to zero. The above equation can be written as

$$\dot{\sigma} = -2(\sigma + p)\frac{\dot{a}}{a}, \quad (17)$$

and using  $\sigma' = \frac{\dot{\sigma}}{\dot{a}}$ , it yields

$$a\sigma' = -2(\sigma + p). \quad (18)$$

The integral solution of Eq.(17) provides the full understanding of WH dynamics given as

$$\ln \frac{a}{a(\tau_0)} = -\frac{1}{2} \int_{\sigma(\tau_0)}^{\sigma} \frac{d\sigma}{\sigma + p(\sigma)}, \quad (19)$$



and can then be inverted to obtain  $\sigma = \sigma(a)$ . Thus, Eq.(10) takes the form

$$\dot{a}^2 + \phi(a) = 0, \quad (20)$$

where  $\phi(a)$  is the potential function

$$\phi(a) = H(a) - [2\pi a\sigma(a)]^2. \quad (21)$$

Taking the derivative of above equation and using (18), we have

$$\phi'(a) = H'(a) + 8\pi^2 a\sigma(a) [\sigma(a) + p(a)]. \quad (22)$$

This potential function is used to discuss the linearized stability analysis of static solution under radial perturbations. For this purpose, we apply Taylor expansion to potential function upto second order around  $a = a_0$  to provides

$$\phi(a) = \phi(a_0) + \phi'(a_0)(a - a_0) + \frac{1}{2}\phi''(a_0)(a - a_0)^2 + O[(a - a_0)^3]. \quad (23)$$

The first derivative of EoS takes the form

$$p'(a) = \sigma'(a) \left[ \omega(1 + \beta)(\sigma(a)^{1+\beta} - L)^{-1-\omega} - \frac{\beta p}{\sigma} \right], \quad (24)$$

which further can be written as

$$\sigma'(a) + 2p'(a) = \sigma'(a) \left[ 1 + 2\omega(1 + \beta)(\sigma(a)^{1+\beta} - L)^{-1-\omega} - \frac{2\beta p(a)}{\sigma(a)} \right]. \quad (25)$$

The stability of static WH solutions needs conditions  $\phi(a_0) = 0 = \phi'(a_0)$ , also stable and unstable WH solutions corresponds to  $\phi''(a_0) > 0$  and  $\phi''(a_0) < 0$ , respectively. It can be easily verified that  $\phi(a_0) = 0 = \phi'(a_0)$  by using Eq.(14) in Eqs.(21) and (22). Now, the second derivative of the potential function with Eq.(25) can be written as

$$\begin{aligned} \phi''(a) &= H''(a) - 8\pi^2 \left\{ [\sigma(a) + 2p(a)]^2 + 2\sigma(a)(\sigma(a) + p(a)) \left[ \left(1 - 2\beta\frac{p}{\sigma}\right) \right. \right. \\ &\quad \left. \left. + 2\omega(1 + \beta)(\sigma^{1+\beta} - L)^{-1-\omega} \right] \right\}, \end{aligned} \quad (26)$$

and using Eq.(14) in the above equation, it leads to

$$\begin{aligned}
\phi''(a_0) = & H''(a_0) + \frac{(\beta - 1)[H'(a_0)]^2}{2H(a_0)} + \frac{H'(a_0)}{a_0} \{1 - 2\omega(1 + \beta) \\
& \times \left[ \left( \frac{\sqrt{H(a_0)}}{2\pi a_0} \right)^{1+\beta} + L \right]^{-1-\omega} \} - \frac{2H(a_0)(1 + \beta)}{a_0^2} \\
& \times \left\{ 1 - 2\omega \left[ \left( \frac{\sqrt{H(a_0)}}{2\pi a_0} \right)^{1+\beta} + L \right]^{-1-\omega} \right\}. \tag{27}
\end{aligned}$$

## 4 Born-Infeld Thin-Shell Wormholes

This section deals with the possible existence of BI thin-shell WHs with GCCG corresponding to various values of  $M$ ,  $Q$ , BI parameter  $b$  of BI black hole and constant  $L$ . Each numerical solution of Eq.(15) for  $a_0$  will represent a BI thin-shell WH. We replace these solutions in Eq.(27) to investigate its stability. For this purpose, the whole region is divided into three parts: if the contour lies in the regions where  $a_0 > r_h$  and  $\phi''(a_0) > 0$  or  $\phi''(a_0) < 0$  will correspond to the stable (light green) or unstable (light yellow) static solution and will be non-physical zone (grey) if  $a_0 \leq r_h$ , where  $r_h$  is event horizon of BI black hole. It is noted that static solutions have an important change around  $\frac{Q_c}{M}$  depending upon the BI parameter  $\frac{b}{M}$ , where  $Q_c$  is the critical charge associated with  $Q$  for which the given metric has no horizon. We have chosen those values of  $\frac{Q_c}{M}$  for plotting for which  $H(r_h) = 0 = H'(r_h)$  (obtained numerically). We find that event horizon radius  $\frac{a_0}{M}$  decreases as charge increases and finally disappear when  $\frac{Q}{M} > \frac{Q_c}{M}$  (Figures 1-6). The results of Eqs.(15) and (27) for different parametric values of  $\frac{b}{M} = 1, 2, 5$ ,  $\beta = 0.2, 1$ , charge and constant  $L$  are summarizes as follows.

- Figure 1, 2, 3, shows static WH solutions corresponding to gas exponent  $\beta = 0.2$ , different values of charge and BI parameter  $\frac{b}{M} = 1, 2, 5$ . We see that there exist one unstable solution for each case corresponding to  $|Q| = 0$  and  $|Q| = 0.7Q_c$  with  $\frac{b}{M} = 1, 2, 5$ . When  $\frac{b}{M} = 1, 2$ , we observe that there are two unstable and one stable solutions corresponds to  $|Q| = 0.9999Q_c$  and two (unstable and stable) solutions corresponds to  $|Q| = 1.1Q_c$ , while for  $\frac{b}{M} = 5$  only stable and unstable

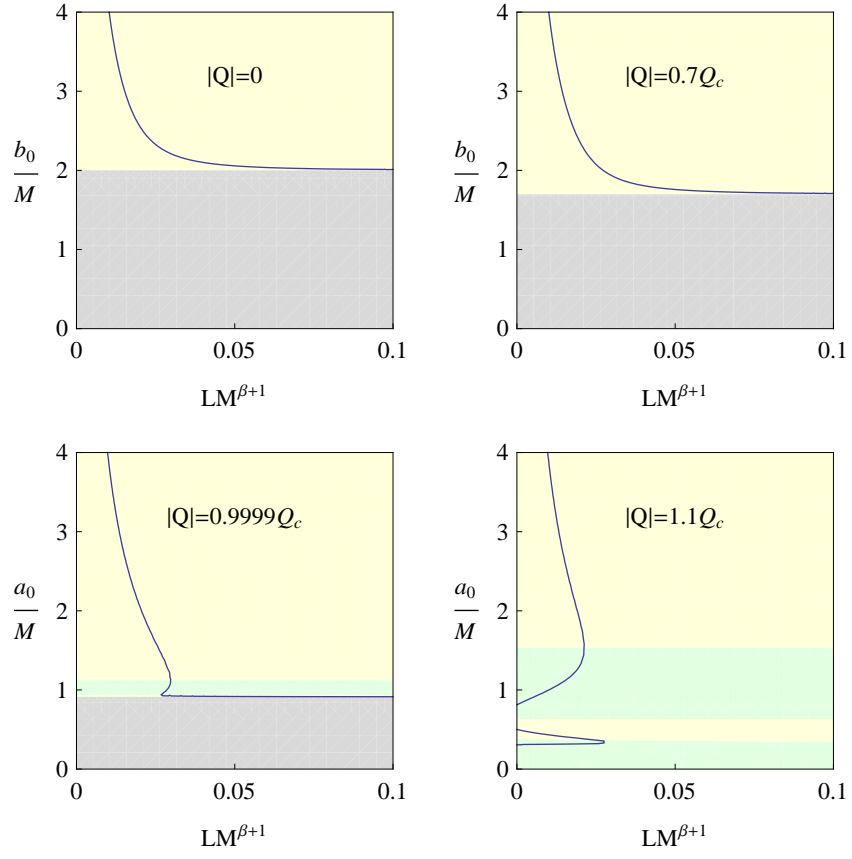


Figure 1: Thin-Shell WHs with Born-Infeld parameter  $\frac{b}{M} = 1$ ,  $L = 0.09$ ,  $\omega = -5$ , gas exponent  $\beta = 0.2$  and distinct values of charge.

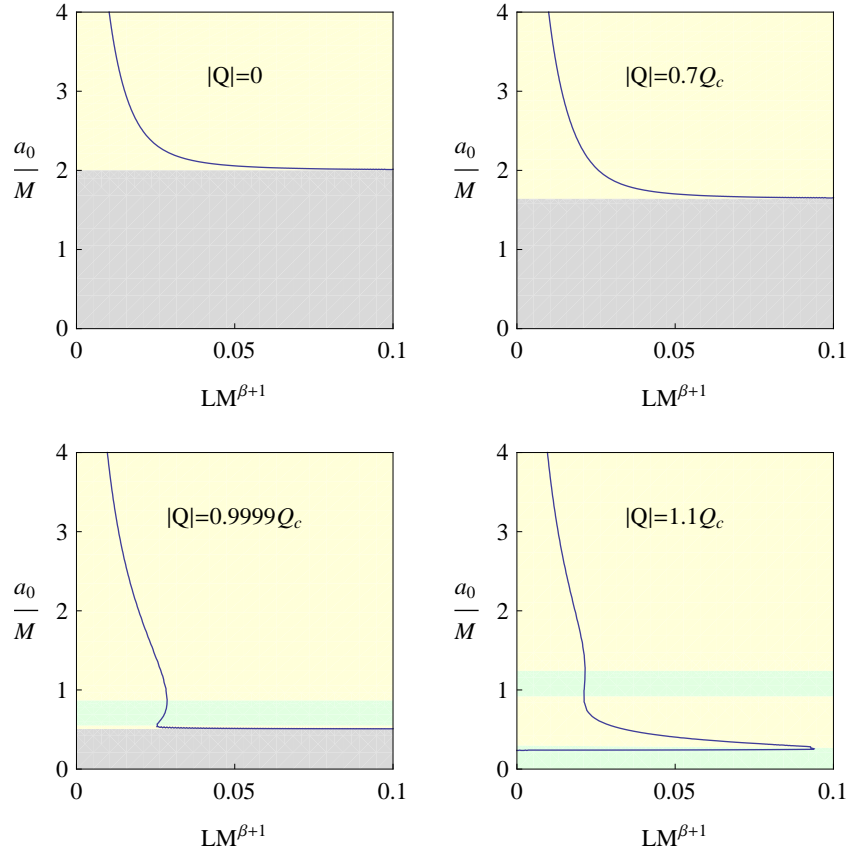


Figure 2: For  $\frac{b}{M} = 2$ ,  $\beta = 0.2$ ,  $L = 0.09$ ,  $\omega = -5$  and distinct values of charge.

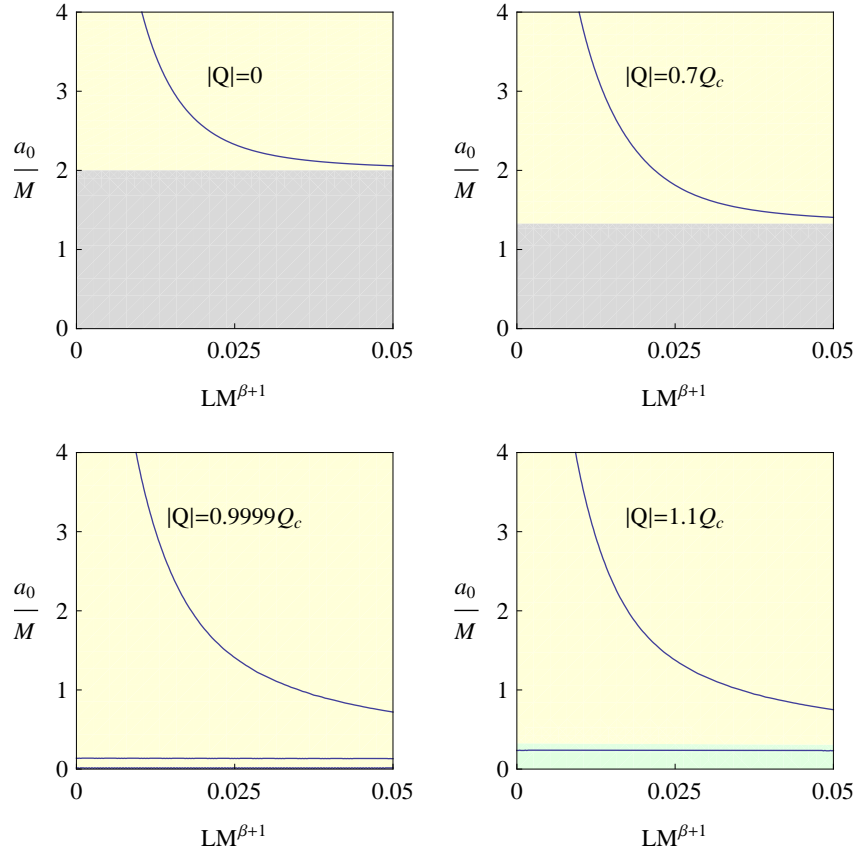


Figure 3: For  $\frac{b}{M} = 5$ ,  $\beta = 0.2$ ,  $L = 0.09$ ,  $\omega = -5$  and distinct values of charge.

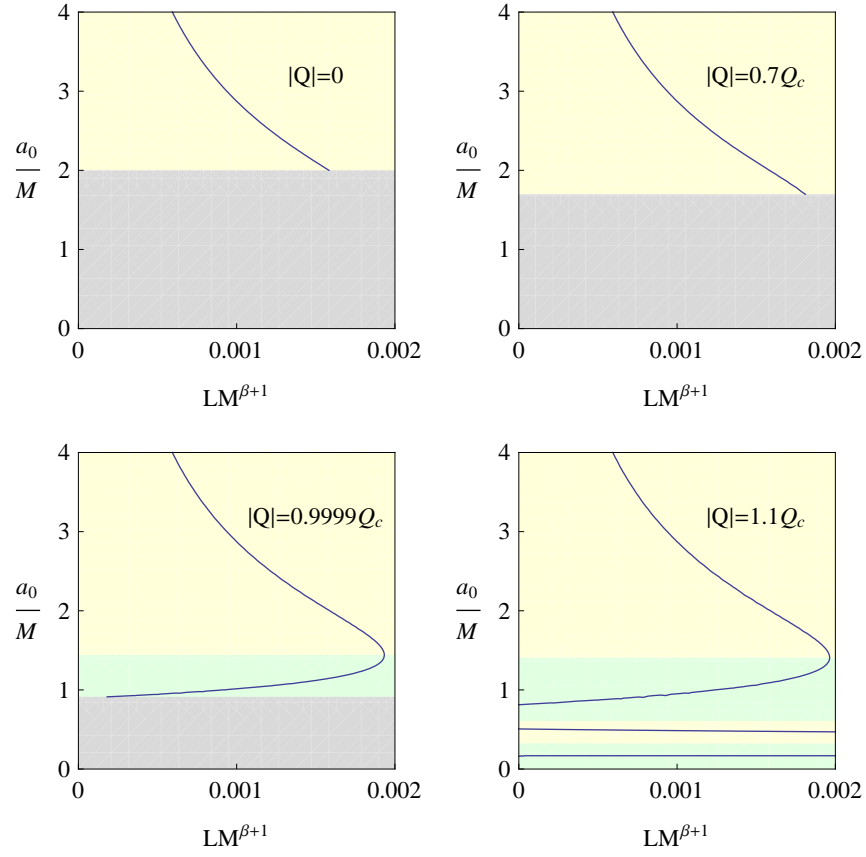


Figure 4: For  $\frac{b}{M} = 1$ ,  $\beta = 1$ ,  $L = 0.09$ ,  $\omega = -5$  and distinct values of charge.

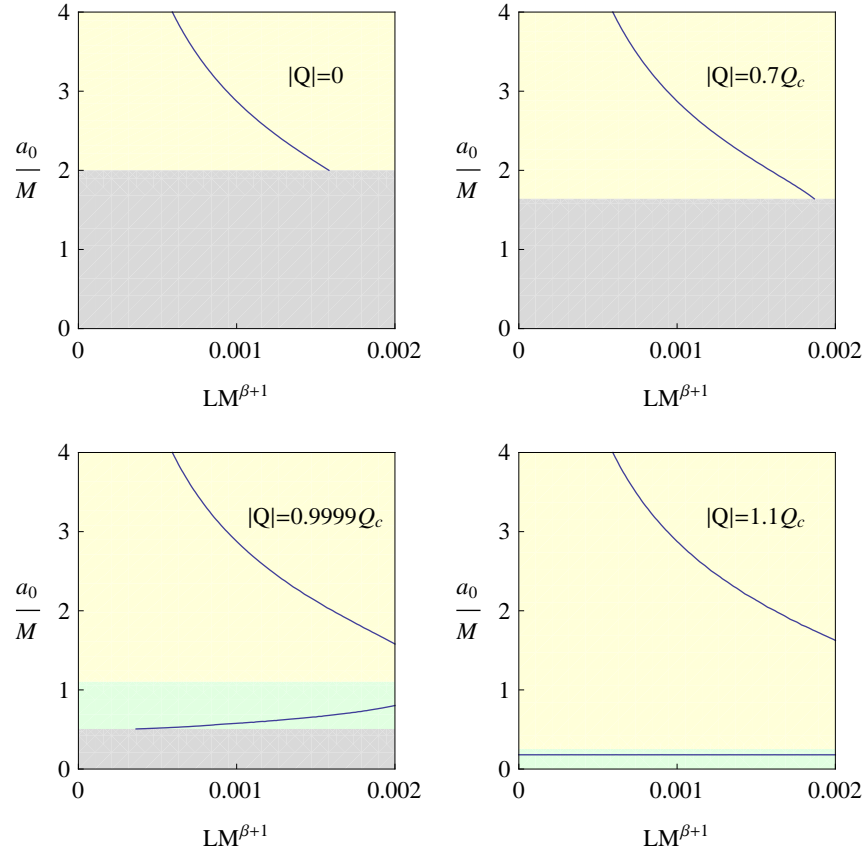


Figure 5: For  $\frac{b}{M} = 2$ ,  $L = 0.09$ ,  $\omega = -5$ ,  $\beta = 1$ , and distinct values of charge.

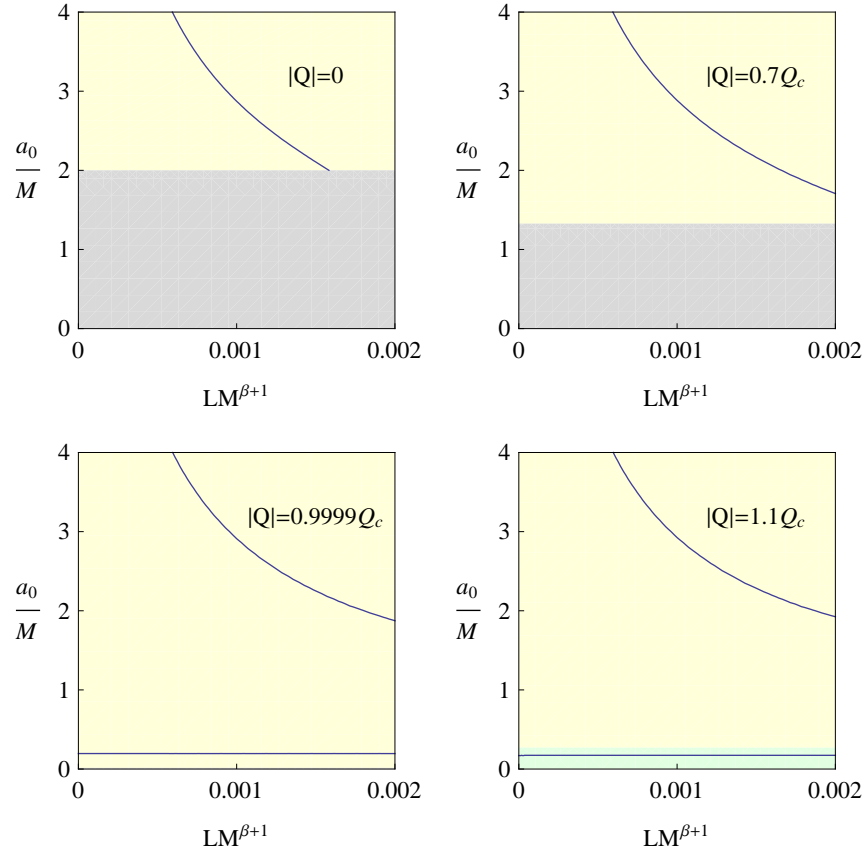


Figure 6: For  $\frac{b}{M} = 5$ ,  $\beta = 1$ ,  $L = 0.09$ ,  $\omega = -5$  and distinct values of charge.



solutions are exist corresponding to  $|Q| = 0.9999Q_c$  and  $|Q| = 1.1Q_c$ , respectively. In each case, the critical charge has different value when  $\beta = 0.2$ , i.e.,  $\frac{Q_c}{M} = 1.02526, 1.10592, 1.148468$  for  $\frac{b}{M} = 1, 2, 5$ , respectively. Moreover, the horizon of the original manifold decreases continuously for increasing value of charge and eventually disappears for large value of  $\frac{b}{M}$  and  $|Q| > Q_c$ , where both unstable and stable solutions are exists.

- Figures 4, 5, 6, represents static WH solutions with  $\beta = 1$ ,  $\frac{b}{M} = 1, 2, 5$  and different values of charge. The possible solutions are similar to the above case for  $|Q| = 0, 0.7Q_c$  and  $\frac{b}{M} = 1, 2, 5$ , while both stable and unstable solutions exist when  $\frac{b}{M} = 1, 2$  and  $|Q| = 0.9999Q_c$ . Also, similar solutions appears to the above case when  $\frac{b}{M} = 1$  and  $|Q| = 1.1Q_c$ , while one less stable and unstable exist when  $\frac{b}{M} = 2$  and  $|Q| = 1.1Q_c$ . However, for  $\frac{b}{M} = 5$ , same solutions are found either we take  $\beta = 0.2$  or  $\beta = 1$  with  $|Q| = 0.9999Q_c$  and  $|Q| = 1.1Q_c$ . Also a similar behavior of horizon radius is observed.

## 5 Discussion and Conclusions

In this work, we have formulated BI thin-shell WHs supported with GCCG and look into their linearize stability analysis via radial perturbations (preserve the symmetry). We have solved evolution equation (15) of static WHs numerically and used in (26) to investigate their stability. We have found static WHs solutions corresponding to different values of charge, gas exponent  $\beta = 0.2, 1$ , BI parameter  $\frac{b}{M} = 1, 2, 5$ ,  $\omega = -5$  and constants involve in the model. The results are shown in Figures 1-6. The solutions in the light (green and yellow) regions are represented as (stable and unstable) solutions respectively.

**Table 1 Comparison of BI Static WH Solutions with GCG, MCG and GCCG EoS**

Value of $\beta$	EoS	$\frac{b}{M} =$	$\frac{ Q }{M} = 0$	$\frac{ Q }{M} = 0.7$	$\frac{ Q }{M} = 0.999$	$\frac{ Q }{M} = 1.1$
$\beta = 0.2$	<i>GCG</i>	1	1U	1U	2U, 1S	2U, 1S
$\beta = 0.2$	<i>MCG</i>	1	1U	1U	2U, 1S	2U, 1S
$\beta = 0.2$	<i>GCCG</i>	1	1U	1U	2U, 1S	2U, 2S
$\beta = 0.2$	<i>GCG</i>	2	1U	1U	2U, 1S	2U, 1S
$\beta = 0.2$	<i>MCG</i>	2	1U	1U	2U, 1S	1U
$\beta = 0.2$	<i>GCCG</i>	2	1U	1U	2U, 1S	2U, 2S
$\beta = 1$	<i>GCG</i>	1	1U	1U	1U, 1S	2U, 1S
$\beta = 1$	<i>MCG</i>	1	1U, 1S	1U, 1S	1U, 1S	1U
$\beta = 1$	<i>GCCG</i>	1	1U	1U	1U, 1S	2U, 2S
$\beta = 1$	<i>GCG</i>	2	1U	1U	1U, 1S	1U
$\beta = 1$	<i>MCG</i>	2	1U, 1S	1U, 1S	1U, 1S	1U
$\beta = 1$	<i>GCCG</i>	2	1U	1U	1U, 1S	1U, 1S
$\beta = 0.2$	<i>GCG</i>	5	1U	1U	1U	1U
$\beta = 0.2$	<i>MCG</i>	5	1U	1U	1U	1U
$\beta = 0.2$	<i>GCCG</i>	5	1U	1U	2U	1U, 1S
$\beta = 1$	<i>GCG</i>	5	1U	1U	1U	1U
$\beta = 1$	<i>MCG</i>	5	1U, 1S	1U, 1S	1U	1U
$\beta = 1$	<i>GCCG</i>	5	1U	1U	2U	1U, 1S

In order to see the role played by the GCCG EoS in the existence and stability of static WH solutions, a comparison of static WH solutions between GCCG, MCG and GCG is given in the table 1. Eiroa and Aguirre (2012) shows that for small values  $b$  and charge, there exists similar results to the Reissner-Nordström case (Eiroa, 2009). However, for large value of  $b$ , the Einstein-Born-Infeld theory deviating from the Einstein-Maxwell theory, the stable regions are disappear and only unstable solutions are possible. Sharif and Azam (2014) extended this work with MCG and found stable as well unstable static WH solutions even for large value of BI parameter.

In this work, we have constructed the BI thin-shell WHs in the vicinity of GCCG and their stability. We can see a similar behavior of static solutions from table 1 when  $|Q|$  is not very close to  $Q_c$  ( $0 \leq |Q| < Q_c$ ) and for different values of charge and BI parameter, i.e., only unstable WH solution exists all considered EoS, except the case of MCG, we have one extra stable solution for  $\beta = 1$  corresponding distinct values of  $b$ . However, we have found one

extra stable static WH solution corresponding to distinct values of  $\beta$  and  $b$  with GCCG when  $|Q| \geq Q_c$ . This fact supports the consistency of results that extra solutions are present for large value of charge with GCCG (Sharif and Azam, 2013d). Thus, it is concluded that GCCG and large value of charge are the most critical factors for the existence of such extra stable WH solutions.

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